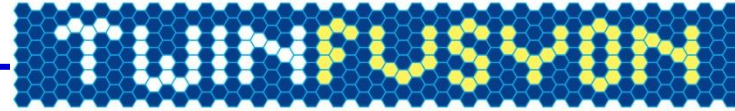


Propagation of Light in Layered Materials

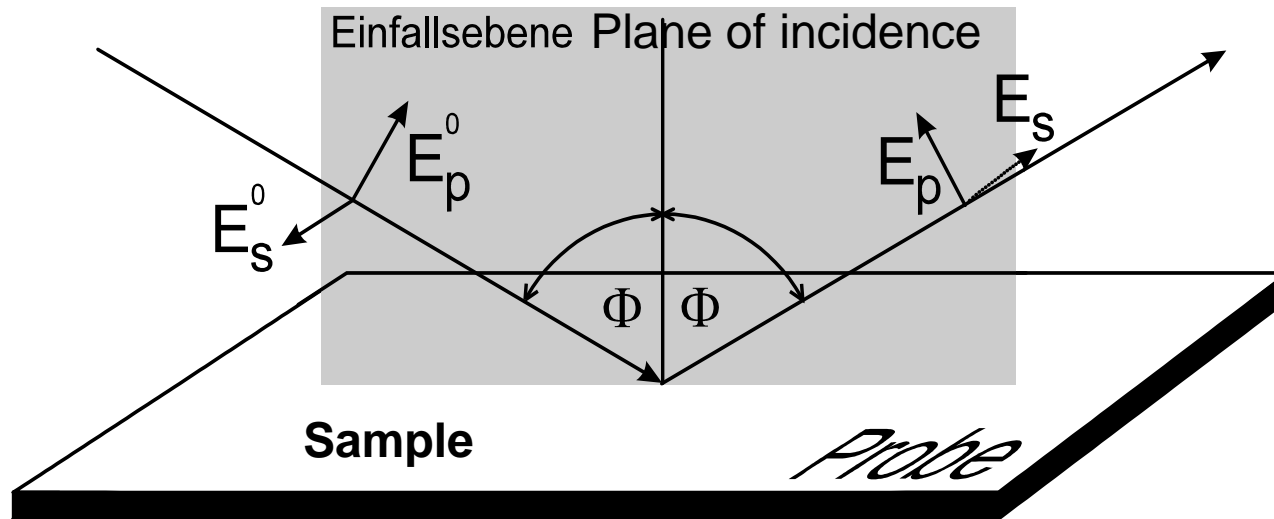
Kurt.Hingerl@jku.at

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& Christian Doppler Lab for Surface Optics
Johannes Kepler University Linz, Austria

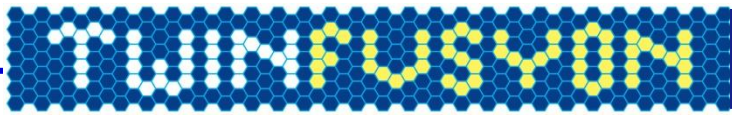


- **Recapitulation of Electrodynamics**
- **2 phase and 3- phase model**
- Transmission (scattering) matrix,
- Virtual interface model

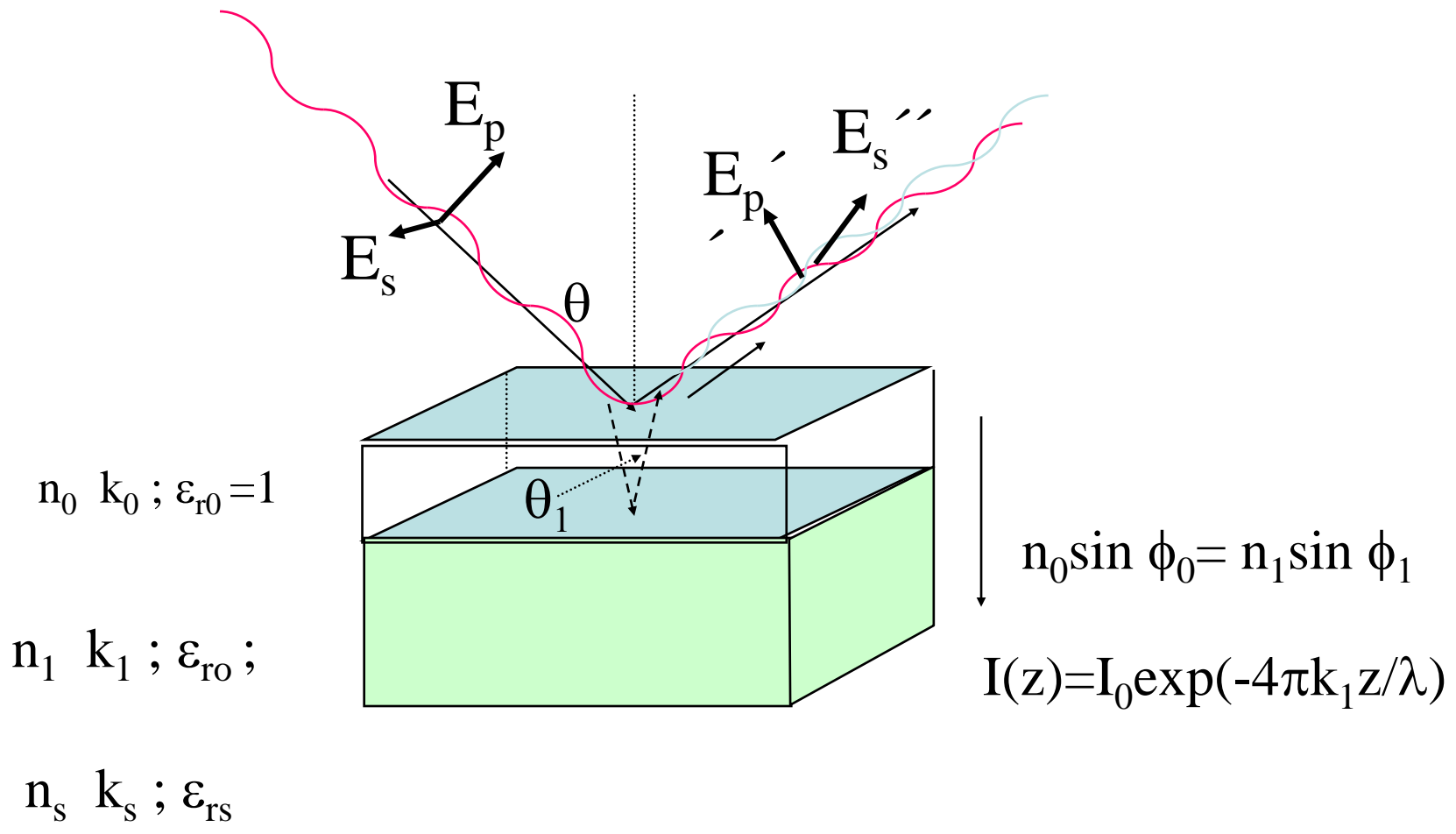
Components of Polarization: 'senkrecht' 'parallel'



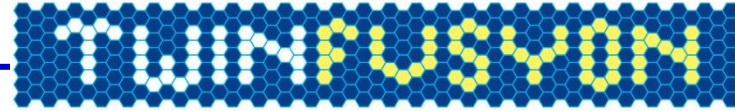
Recap II: Polarization, Reflection, Refraction



3-phase model



Maxwell's equations in vacuum and matter:



$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \quad \text{microscopic fields}$$

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right) \quad \nabla \cdot \mathbf{B} = 0$$

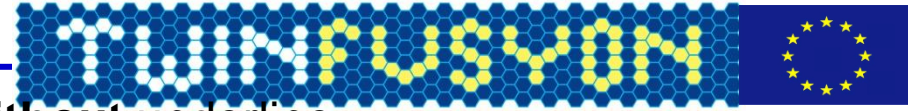
E and **H** are electric and magnetic field, **D** and **B** are displacement and magnetic induction fields; **E** and **B** are observable fields,

Bound charges and currents are put into the macroscopic fields **E** **B**

$$\nabla \cdot \underline{\mathbf{E}} = \frac{1}{\epsilon_0} (\rho_f + \rho_b) = \frac{\rho_f}{\epsilon_0} - \frac{\nabla \cdot \mathbf{P}}{\epsilon_0} \quad \dots \mathbf{P} = \frac{1}{\Delta V} \sum \mathbf{p}$$

$$\nabla \times \underline{\mathbf{B}} = \mu_0 \left(\mathbf{J}_f + \mathbf{J}_b + \epsilon_0 \frac{\partial \underline{\mathbf{E}}}{\partial t} \right) = \mu_0 \left(\mathbf{J}_f + \frac{\partial \mathbf{P}}{\partial t} + \nabla \times \mathbf{M} + \epsilon_0 \frac{\partial \underline{\mathbf{E}}}{\partial t} \right)$$

Linear Material properties



E all fields from now on macroscopic fields without underline, most general linear approach

$$\mathbf{D}(\mathbf{r}, t) = \varepsilon_0 \vec{\varepsilon}(\mathbf{r}, t) \mathbf{E}(\mathbf{r}, t) = \varepsilon_0 \vec{\varepsilon}(t) \mathbf{E}(\mathbf{r}, t) = \varepsilon_0 \mathbf{E}(\mathbf{r}, t) + \mathbf{P}(\mathbf{r}, t) = \varepsilon_0 (1 + \chi(\mathbf{r}, t)) \mathbf{E}(\mathbf{r}, t)$$

$$\vec{\varepsilon}(\mathbf{r}, \omega) \rightarrow \vec{\varepsilon}(\omega) \rightarrow \varepsilon(\omega)$$

$$\mathbf{B}(\mathbf{r}, t) = \mu \mathbf{H}(\mathbf{r}, t) = \mu_0 (1 + \chi_m) \mathbf{H}(\mathbf{r}, t) \text{ for optical frequencies } \mathbf{B}(\mathbf{r}, t) = \mu_0 \mathbf{H}(\mathbf{r}, t)$$

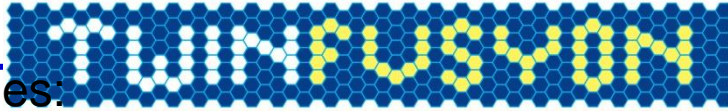
If ε tensor- see lecture on generalized ellipsometry, if ε is a function of \mathbf{r} , then solve a more complicated diff. equation, $\varepsilon(t)$ real \Rightarrow FT: $\varepsilon(\omega) = \varepsilon(-\omega)$

Redefine ME in macroscopic fields:

$$\begin{aligned} \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} & \nabla \cdot \mathbf{D} &= \rho_f \\ \nabla \times \mathbf{H} &= \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t} & \nabla \cdot \mathbf{B} &= 0 \end{aligned}$$

$$\begin{aligned} \varepsilon &= \varepsilon_0 (1 + \chi) & \mathbf{D} &= \varepsilon \mathbf{E} \\ \mu &= \mu_0 (1 + \chi_m) & \mathbf{B} &= \mu \mathbf{H} \end{aligned}$$

Dispersion relation and continuity at interfaces



No free charges or currents at optical frequencies:

$$\Delta \mathbf{E} - \mu \epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \quad \mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t + \phi_e)}$$

$$\tilde{n}^2 = (n - ik)^2 = \mu \epsilon c^2 = \frac{\mu}{\mu_0} \frac{\epsilon}{\epsilon_0} \quad k = \tilde{n} \frac{\omega}{c} \quad \text{Dispersion relation: } k^2 = \mu \epsilon \omega^2$$

- Normal component of $\mathbf{D} = \epsilon \mathbf{E}$ continuous
- Normal component of \mathbf{B} continuous
- Tangential component of \mathbf{E} continuous
- Tangential component of $\mathbf{H} = \mathbf{B}/\mu$ continuous

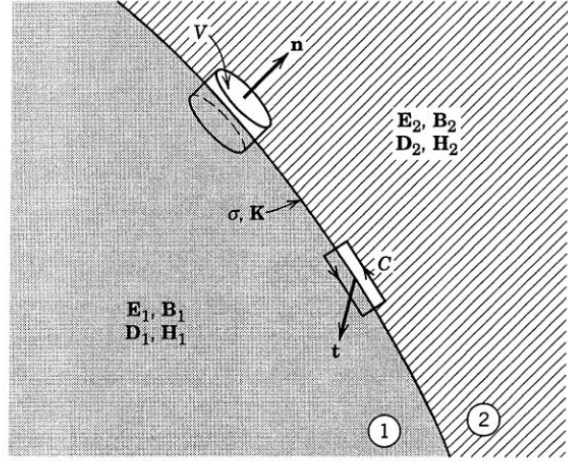


Figure I.4 Schematic diagram of boundary surface (heavy line) between different media. The boundary region is assumed to carry idealized surface charge and current densities σ and \mathbf{K} . The volume V is a small pillbox, half in one medium and half in the other, with the normal \mathbf{n} to its top pointing from medium 1 into medium 2. The rectangular contour C is partly in one medium and partly in the other and is oriented with its plane perpendicular to the surface so that its normal \mathbf{t} is tangent to the surface.

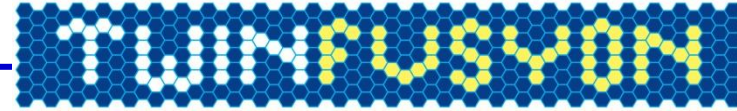
$$\oiint_S \mathbf{D} \cdot \mathbf{n} dA = (\mathbf{D}_1 - \mathbf{D}_2) \cdot \mathbf{n} \Delta A = \epsilon (\mathbf{E}_1 - \mathbf{E}_2) \cdot \mathbf{n} \Delta A = \rho = 0$$

$$(\mathbf{B}_1 - \mathbf{B}_2) \cdot \mathbf{n} = 0 \quad \epsilon (\mathbf{E}_1 - \mathbf{E}_2) \cdot \mathbf{n} = 0$$

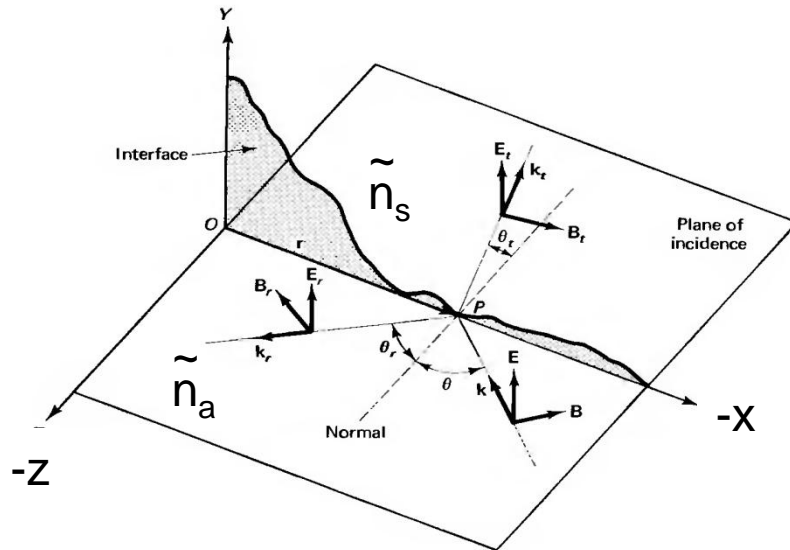
$$\oint_C \mathbf{E} d\mathbf{l} = (\mathbf{t} \times \mathbf{n})(\mathbf{E}_2 - \mathbf{E}_1) \cdot \Delta \mathbf{l} = 0 = - \iint_{S'} \frac{\partial \mathbf{B}}{\partial t} \cdot \mathbf{n} dA$$

$$\mathbf{n} \times (\mathbf{E}_2 - \mathbf{E}_1) = 0 \quad \mathbf{n} \times (\mathbf{H}_2 - \mathbf{H}_1) = 0$$

Continuity II : waves from beneath



s-, TE, σ polarization

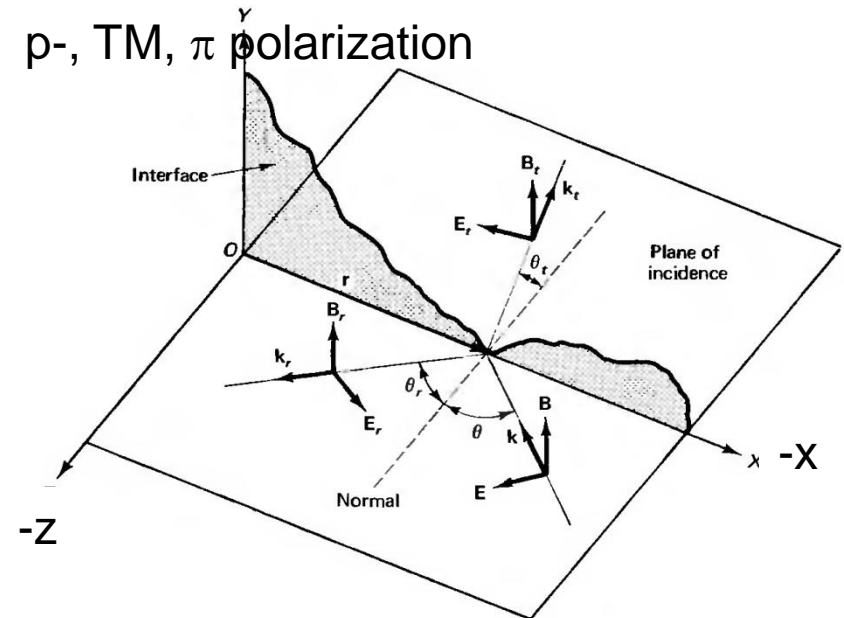


$$\mathbf{k} = (k_x, 0, k_z) = \tilde{n}_a \frac{\omega}{c} (\sin \theta, 0, \cos \theta)$$

$$\mathbf{k}^r = (k_x^r, k_y^r, k_z^r) \quad k_x^{r2} + k_y^{r2} + k_z^{r2} = \left(\tilde{n}_a \frac{\omega}{c} \right)^2$$

$$\mathbf{k}^t = (k_x^t, k_y^t, k_z^t) \quad k_x^{t2} + k_y^{t2} + k_z^{t2} = \left(\tilde{n}_s \frac{\omega}{c} \right)^2$$

p-, TM, π polarization



$$\mathbf{E}_{\text{tang}} e^{ixk_x} + \mathbf{E}_{\text{tang}}^r e^{i(xk_x^r + yk_y^r)} = \mathbf{E}_{\text{tang}}^t e^{i(xk_x^t + yk_y^t)}$$

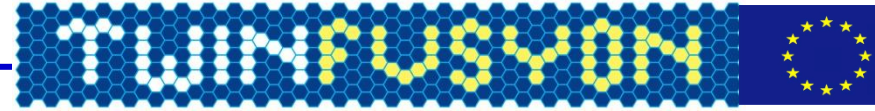
$$\mathbf{H}_{\text{tang}} e^{ixk_x} + \mathbf{H}_{\text{tang}}^r e^{i(xk_x^r + yk_y^r)} = \mathbf{H}_{\text{tang}}^t e^{i(xk_x^t + yk_y^t)}$$

Phases have to be the same at $z=0$:

$$k_y^r = k_y^t = 0, \quad k_x^r = k_x \quad k_x^t = k_x$$

$$\boxed{\tilde{n} \sin \theta = \tilde{n}' \sin \theta'}$$

Fresnel's formulas I : reflection for s-, TE, σ pol.



$$\mathbf{E}_y + \mathbf{E}_y^r = \mathbf{E}_y^t \quad (1)$$

$$\left(\mathbf{B}_x + \mathbf{B}_x^r\right) = \frac{\mu_a}{\mu_s} \mathbf{B}_x^t \quad \frac{1}{\omega} \left(\mathbf{k} \times \mathbf{E}_y\right)_x + \left(\mathbf{k}^r \times \mathbf{E}_y^r\right)_x = \frac{\mu_a}{\omega \mu_s} \left(\mathbf{k}^t \times \mathbf{E}_y^t\right)_x \quad (2)$$

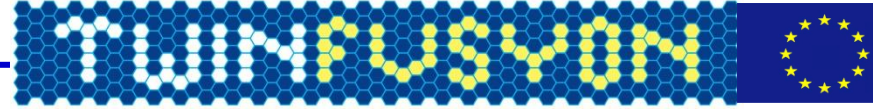
$$\left(\mathbf{k}_z \mathbf{E}_y\right) - \left(\mathbf{k}_z \mathbf{E}_y^r\right) = \frac{\mu_a}{\mu_s} \left(\mathbf{k}_z^t \mathbf{E}_y^t\right) \quad (2a)$$

$$\mathbf{E}^t = \frac{2k_z \mu_s}{k_z \mu_s + k_z^t \mu_a} \mathbf{E} = t_s \mathbf{E} \quad \mathbf{E}^r = \frac{k_z \mu_s - k_z^t \mu_a}{k_z \mu_s + k_z^t \mu_a} \mathbf{E} = r_s \mathbf{E} \quad \mu_s = \mu_a = 1$$

$$r_s = \frac{\tilde{n}_a \cos \theta - \tilde{n}_s \cos \theta_s}{\tilde{n}_a \cos \theta + \tilde{n}_s \cos \theta_s} = \frac{\sqrt{\epsilon_a - \epsilon_a \sin^2(\theta)} - \sqrt{\epsilon_s - \epsilon_a \sin^2(\theta)}}{\sqrt{\epsilon_a - \epsilon_a \sin^2(\theta)} + \sqrt{\epsilon_s - \epsilon_a \sin^2(\theta)}}$$

$$t_s = \frac{2\tilde{n}_a \cos \theta}{\tilde{n}_a \cos \theta + \tilde{n}_s \cos \theta_s} = \frac{2\sqrt{\epsilon_a - \epsilon_a \sin^2(\theta)}}{\sqrt{\epsilon_a - \epsilon_a \sin^2(\theta)} + \sqrt{\epsilon_s - \epsilon_a \sin^2(\theta)}}$$

Fresnel's formulas II : reflection for p-, TM, π pol.



$$\mathbf{B}_y + \mathbf{B}_y^r = \frac{\mu_a}{\mu_s} \mathbf{B}_y^t \quad (1)$$

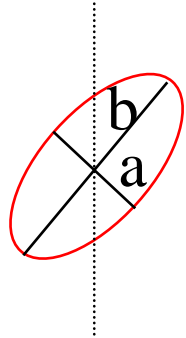
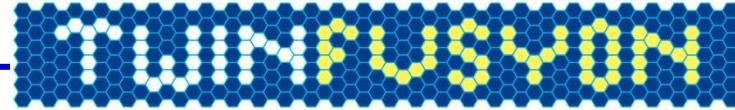
$$\mathbf{E}_x + \mathbf{E}_x^r = \mathbf{E}_x^t \quad \frac{k_z}{\mu_a \epsilon_a \omega} \mathbf{B}_y - \frac{k_z}{\mu_a \epsilon_a \omega} \mathbf{B}_y^r = \frac{k_z^t}{\mu_s \epsilon_s \omega} \mathbf{B}_y^t \quad (2)$$

$$\mathbf{B}^r = \frac{\epsilon_s k_z - \epsilon_a k_z^t}{\epsilon_s k_z + \epsilon_a k_z^t} \mathbf{E} = r_p \mathbf{B} \quad \mathbf{B}^t = \frac{2 \mu_s \epsilon_s k_z}{\mu_a (\epsilon_s k_z + \epsilon_a k_z^t)} \mathbf{B} = t_p^B \mathbf{B}$$

$$r_p = \frac{\tilde{n}_s \cos \theta - \tilde{n}_a \cos \theta_s}{\tilde{n}_s \cos \theta + \tilde{n}_a \cos \theta_s} = \frac{\epsilon_s \sqrt{\epsilon_a - \epsilon_a \sin^2(\theta)} - \epsilon_a \sqrt{\epsilon_s - \epsilon_a \sin^2(\theta)}}{\epsilon_s \sqrt{\epsilon_a - \epsilon_a \sin^2(\theta)} + \epsilon_a \sqrt{\epsilon_s - \epsilon_a \sin^2(\theta)}}$$

$$t_p = \frac{2 \tilde{n}_a \cos \theta_s}{\tilde{n}_s \cos \theta + \tilde{n}_a \cos \theta_s} = \frac{2 \epsilon_a \sqrt{\epsilon_s - \epsilon_a \sin^2(\theta)}}{\epsilon_s \sqrt{\epsilon_a - \epsilon_a \sin^2(\theta)} + \epsilon_a \sqrt{\epsilon_s - \epsilon_a \sin^2(\theta)}}$$

2 Phase System can be solved analytically

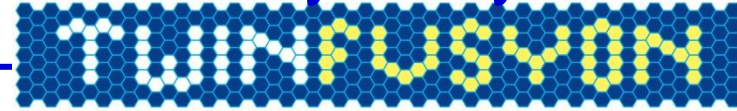


Ψ, Δ (ellipsometric angles)

$$\rho = \frac{r_p}{r_s} = \frac{\sin^2(\theta) - \cos(\theta) \sqrt{\frac{\epsilon_s}{\epsilon_a} - \sin^2(\theta)}}{\sin^2(\theta) + \cos(\theta) \sqrt{\frac{\epsilon_s}{\epsilon_a} - \sin^2(\theta)}} =: \tan \psi e^{i\Delta}$$

$$\frac{\epsilon_s}{\epsilon_a} = \sin^2(\theta) + \sin^2(\theta) \tan^2(\theta) \frac{(1 - \rho)^2}{(1 + \rho)^2}$$

3 Phase System cannot be solved analytically

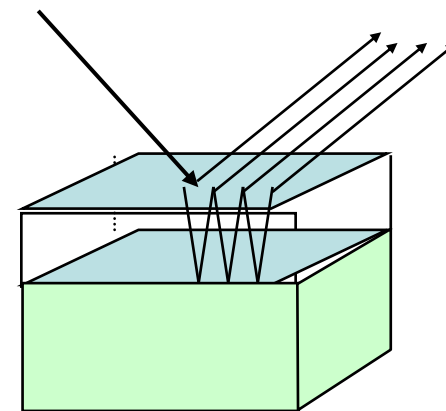
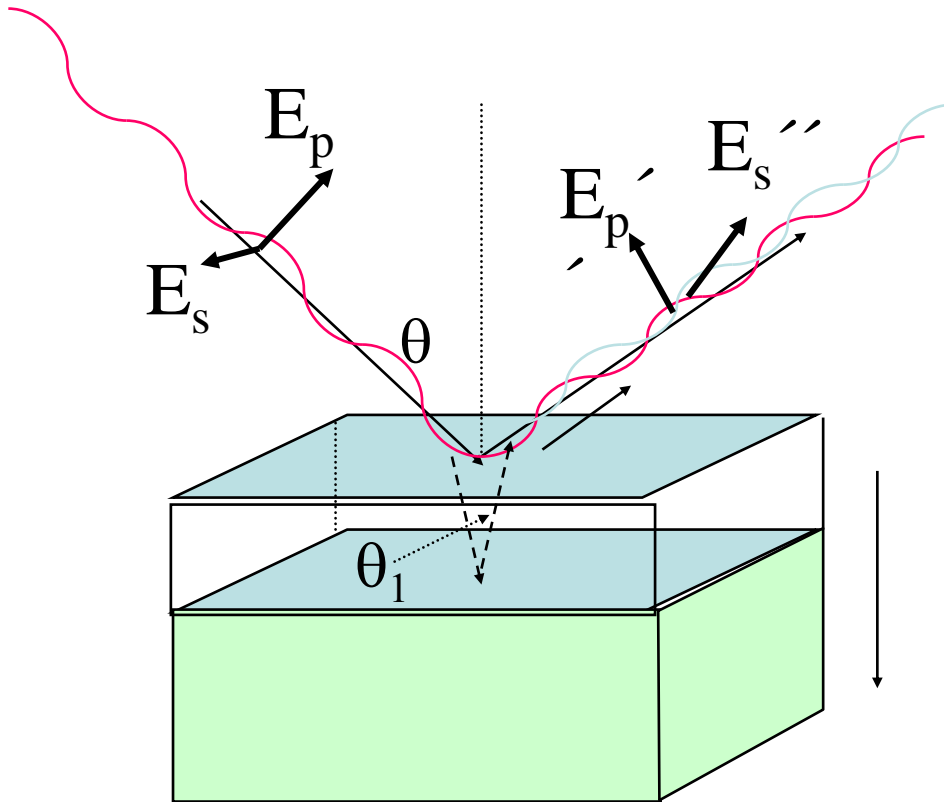


However, ε_a , ε_o , ε_s , and d_o by simulation respectively / fitting is always possible and highly accurate (multiple solutions).

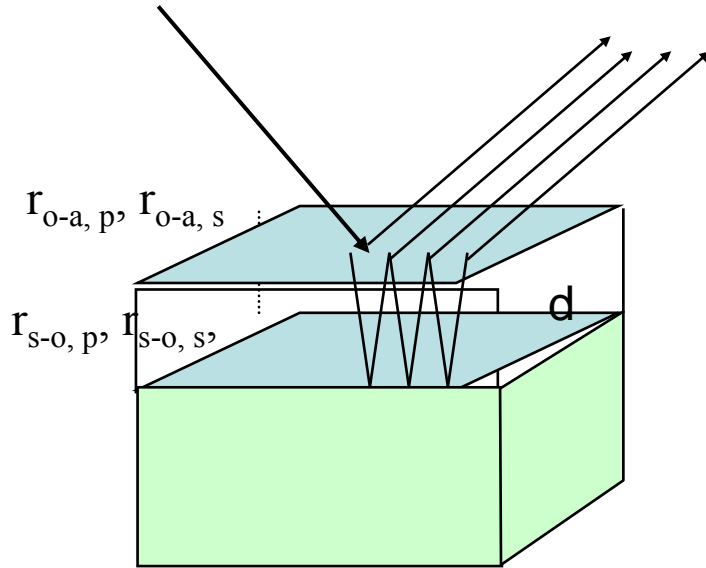
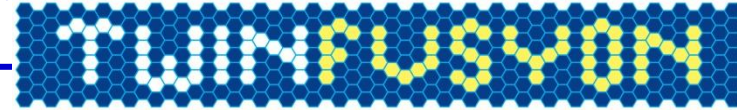
The measured value is still ρ !

Techniques:

- 3 Phase system (extension to more layers by continuous fractions)
- Scattering Matrix (S-matrix)
- T-matrix



3 Phase System



For both polarizations it can be shown:

$$r_{o-a} = -r_{a-o}$$

$$t_{o-a} = (1 - r_{a-o}^2) / t_{a-o}$$

The phase acquired by passing the overlayer back and forth:

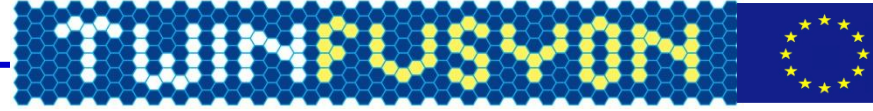
$$Z = e^{i4\pi (d_o/\lambda)\tilde{n}_o \cos\theta_o} = e^{2i(\omega/c)\sqrt{\epsilon_s - \epsilon_a \sin^2(\theta)}}$$

For s- and p-pol therefore:

$$r_{\text{all}} = r_{a-o} + t_{a-o} t_{o-a} r_{o-s} Z + t_{a-o} t_{o-a} r_{o-a} r_{o-s}^2 Z^2 + t_{a-o} t_{o-a} r_{o-a}^2 r_{o-s}^3 Z^3 + \dots$$

$$r_{\text{all}} = r_{a-o} + \frac{t_{a-o} t_{o-a} r_{o-s} Z}{1 - r_{o-a} r_{o-s} Z} = \frac{r_{a-o} + r_{o-s} Z}{1 + r_{a-o} r_{o-s} Z}$$

3 layer: ambient / overlayer / substrate



$$\rho_{\text{all}} = \frac{r_{\text{all,p}}}{r_{\text{all,s}}} = \tan \psi e^{i\Delta} = \frac{r_{\text{a-o,p}} + r_{\text{o-s,p}} Z}{1 + r_{\text{a-o,p}} r_{\text{o-s,p}} Z} / \frac{r_{\text{a-o,s}} + r_{\text{o-s,s}} Z}{1 + r_{\text{a-o,s}} r_{\text{o-s,s}} Z}$$

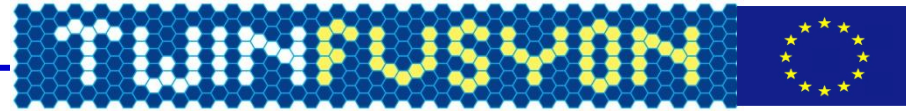
Thin layer expansion:

$$r_{\text{all,p}} = r_{\text{a-s,p}} \left(1 + \frac{d 4\pi i (\dot{\epsilon}_o - \dot{\epsilon}_s) \cos \theta \sqrt{\dot{\epsilon}_a} (\dot{\epsilon}_a (\dot{\epsilon}_o + \dot{\epsilon}_s) \sin^2(\theta) - \dot{\epsilon}_o \dot{\epsilon}_s)}{\lambda_0 \dot{\epsilon}_o (\dot{\epsilon}_s^2 \cos^2(\theta) + \dot{\epsilon}_a (\dot{\epsilon}_a \sin^2(\theta) - \dot{\epsilon}_s))} \right)$$

$$r_{\text{all,s}} = r_{\text{a-s,s}} \left(1 - \frac{d 4\pi i (\dot{\epsilon}_o - \dot{\epsilon}_s) \cos(\theta) \sqrt{\dot{\epsilon}_a}}{\lambda_0 (\dot{\epsilon}_s - \dot{\epsilon}_a)} \right)$$

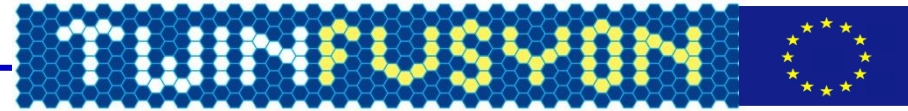
$$\langle \epsilon \rangle = \frac{\epsilon_s}{\epsilon_a} + d \frac{4\pi i \sqrt{\epsilon_a} \epsilon_s (\epsilon_s - \epsilon_o) (\epsilon_o - \epsilon_a)}{\lambda \epsilon_o (\epsilon_s - \epsilon_a)} \sqrt{\frac{\epsilon_s}{\epsilon_a} - \sin^2 \varphi_a}$$

Contents

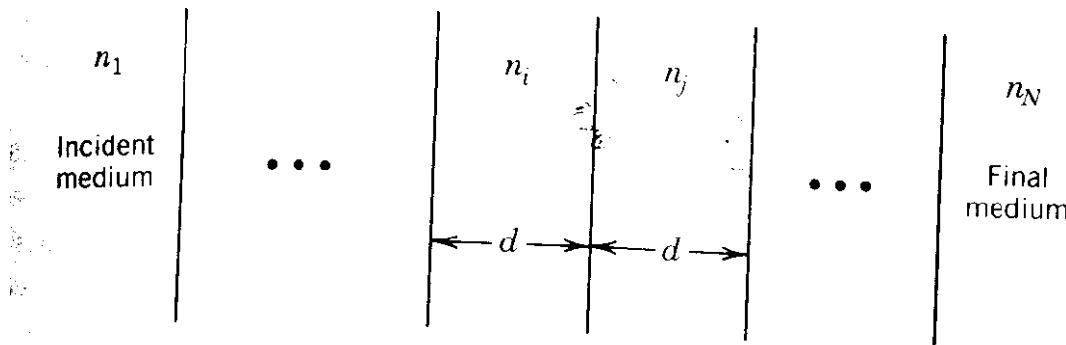


- Recapitulation of Electrodynamics
- 2 phase and 3- phase model
- **Transmission (scattering) matrix,**
- Virtual interface model

Transmission and Scattering matrices:



4 layers: recursive fractions:
$$\frac{r_{a-o,p} + r_{o-s,p} Z}{1 + r_{a-o,p} r_{o-s,p} Z}$$



For s- as well as for p- polarization:

$$E_{rj} = E'_{ri} t_{ij} + E_{lj} r_{ji}$$

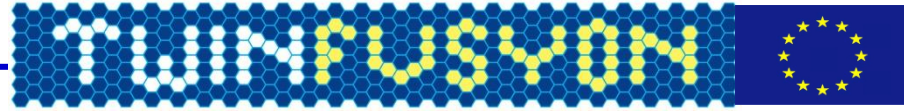
$$E'_{li} = E_{lj} t_{ji} + E'_{ri} r_{ij}$$

Using the relations for the Fresnel coefficients:

Instead: use linear relationship, including the phases: Light is impinging on the structure with N layers from the left. The *i*th layer is the one left of the *j*th. Fields on the left side of each layer are unprimed, on the right side are primed:

E_{rj} is the rightward propagating field on the left side of the *j*th layer (but on the right side of the interface), E'_{lj} is the leftward propagating field in the *j*th layer.

Transmission and Scattering matrices II:



$$\mathbf{E}'_{li} = \frac{1}{t_{ij}} \left(\mathbf{E}_{lj} + r_{ij} \mathbf{E}_{rj} \right)$$

$$\mathbf{E}'_{ri} = \frac{1}{t_{ij}} \left(r_{ij} \mathbf{E}_{lj} + \mathbf{E}_{rj} \right)$$

$$\mathbf{E}'_i = \begin{pmatrix} \mathbf{E}'_{li} \\ \mathbf{E}'_{ri} \end{pmatrix}, \mathbf{E}_j = \begin{pmatrix} \mathbf{E}_{lj} \\ \mathbf{E}_{rj} \end{pmatrix}, \mathbf{H}_{ij} = \frac{1}{t_{ij}} \begin{pmatrix} 1 & r_{ij} \\ r_{ij} & 1 \end{pmatrix}$$

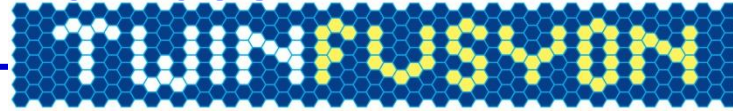
Propagation within one layer:

$$\mathbf{E}'_{rj} = \mathbf{E}_{rj} e^{i\beta_j} = \mathbf{E}_{rj} e^{2i(\omega/c)\sqrt{\epsilon_s - \epsilon_a \sin^2(\theta)}}$$

$$\mathbf{E}_{lj} = \mathbf{E}'_{lj} e^{i\beta_j} = \mathbf{E}'_{lj} e^{2i(\omega/c)\sqrt{\epsilon_s - \epsilon_a \sin^2(\theta)}}$$

$$\mathbf{L}_j = \begin{pmatrix} e^{i\beta_j} & 0 \\ 0 & e^{i\beta_j} \end{pmatrix}$$

Transmission and Scattering matrices III:



$$\mathbf{E}_j = \mathbf{L}_j \mathbf{E}'_j \quad \mathbf{E}_N = \begin{pmatrix} 0 \\ \mathbf{E}_{rN} \end{pmatrix}$$

$$\mathbf{E}_{N-1} = \mathbf{L}_{N-1} \mathbf{H}_{N-1,N} \mathbf{E}_N$$

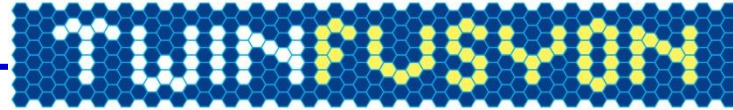
$$\mathbf{E}'_1 = \left(\mathbf{H}_{1,2} \mathbf{L}_{2,\dots} \mathbf{L}_{N-1} \mathbf{H}_{N-1,N} \right) \mathbf{E}_N$$

$$\mathbf{E}'_1 = \mathbf{S}_{1,N} \mathbf{E}_N$$

Matrix \mathbf{S} contains all combined effects of arbitrary many layers including multiple reflections. Per definition, it is related to the total reflection and transmission coefficient

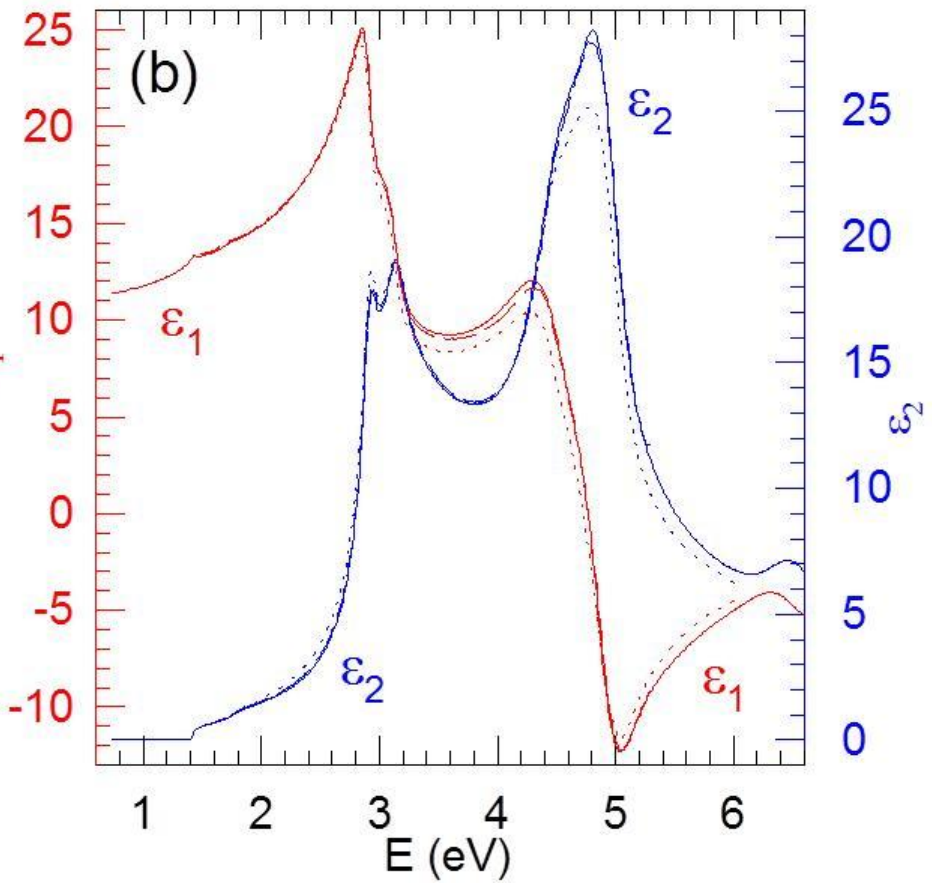
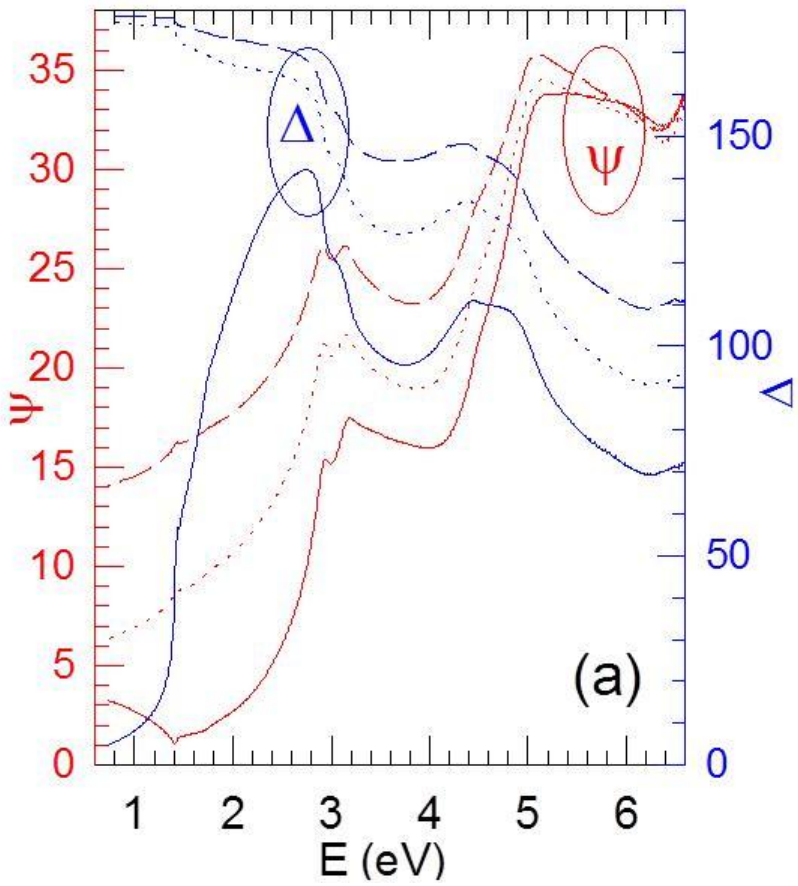
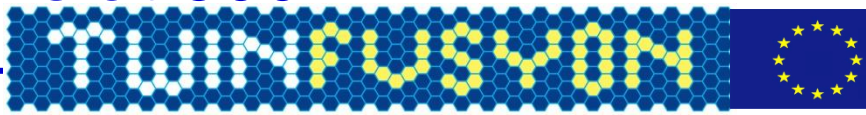
$$\begin{aligned} \begin{pmatrix} \mathbf{E}'_{11} \\ \mathbf{E}'_{r1} \end{pmatrix} &= \begin{pmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} \\ \mathbf{S}_{21} & \mathbf{S}_{22} \end{pmatrix} \begin{pmatrix} 0 \\ \mathbf{E}_{rN} \end{pmatrix} \\ &= \begin{pmatrix} \mathbf{S}_{12} \mathbf{E}_{rN} \\ \mathbf{S}_{22} \mathbf{E}_{rN} \end{pmatrix} \\ r_{1N} &= \frac{\mathbf{E}'_{11}}{\mathbf{E}'_{r1}} = \frac{\mathbf{S}_{12}}{\mathbf{S}_{22}} \\ t_{1N} &= \frac{\mathbf{E}_{rN}}{\mathbf{E}'_{r1}} = \frac{1}{\mathbf{S}_{22}} \end{aligned}$$

Example: Optical Constants of GaAs:



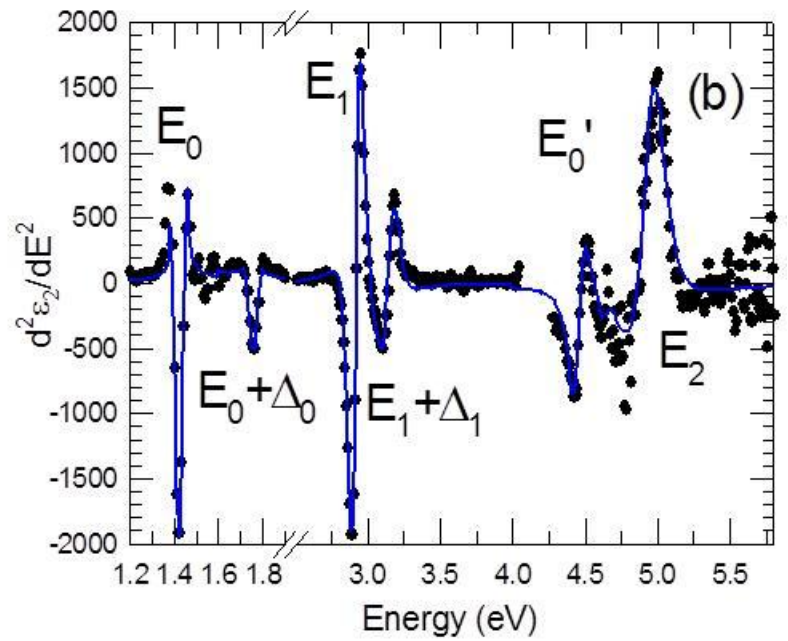
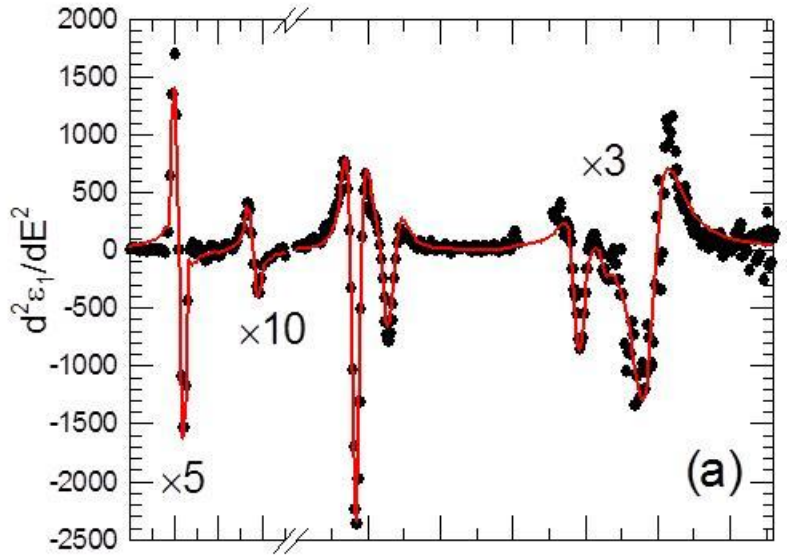
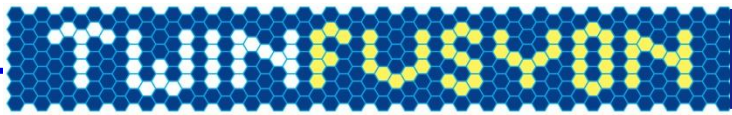
- Below the band gap (0.04 to 1.4 eV):
 - Absorption coefficient from transmission measurements (bulk)
 - Refractive index from prism refraction measurements
 - Both are very accurate, see Blakemore (JAP **53**, R123, 1982).
- Near gap (1.2 to 1.8 eV):
 - n from reflection measurements of bulk samples
 - k from transmission measurements through thin films
 - Good accuracy, see Sell (1974) and Sturge (1962)
- Above the band gap (2 to 6 eV):
 - Kramers-Kronig transform of reflectance data is not accurate.
 - Spectroscopic Ellipsometry: Aspnes and Studna (1984): Rotating-analyzer ellipsometer
 - Rotating-compensator ellipsometer

Dielectric Function of Bulk GaAs at 300 K



(a) Ellipsometric angles at three angles of incidence (65, 70, 75 degrees).
(b) Differences between solid and dotted: oxide coverage (5Å).

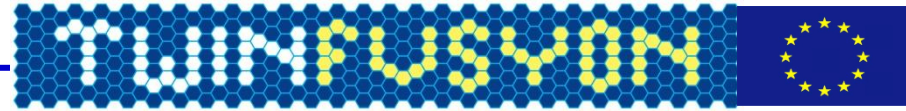
Critical-point parameters of Bulk GaAs at 300 K



- Derivatives related to band structure and critical points (CPs)
- Critical-point parameters:
 A : amplitude
 E : transition energy
 Γ : broadening
 ϕ : phase angle
 n : dimension of CP
- Solid lines show fit to data using analytical lineshapes.
- Good agreement with Lautenschlager *et al.* (1987).

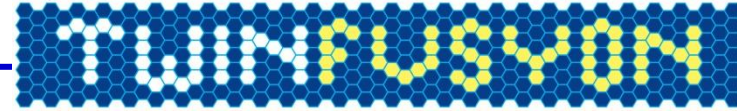
$$\varepsilon(\hbar\omega) = A(\hbar\omega)^{-2} \exp(i\phi)(\hbar\omega - E + i\Gamma)^n$$

Contents



- Recapitulation of Electrodynamics
- 2 phase and 3- phase model
- Transmission (scattering) matrix
- **Virtual interface model**

Virtual Interface Model: Thin layer expansion



During growth /etching we have a time dependent signal: either $r_{\text{all}}(t)$ or $\langle \epsilon \rangle(t)$;

Is it possible –assuming we know the growth rate- to determine the dielectric function of the currently growing layer with dielectric function ϵ_o for any „**virtual**“ substrate? Answer ... Dave Aspnes 1992, 1993, JOSA 10(5), 974

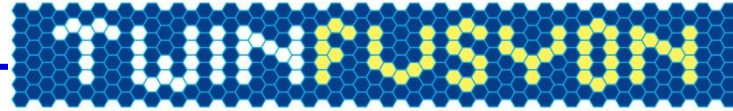
Exact:
$$r_{\text{all},s} = r_{\text{a-s},s} \left(1 - \frac{d \, 4\pi i (\dot{\epsilon}_o - \dot{\epsilon}_s) \cos(\theta) \sqrt{\dot{\epsilon}_a}}{\lambda_o (\dot{\epsilon}_s - \dot{\epsilon}_a)} \right) \Rightarrow \dot{\epsilon}_o = \dot{\epsilon}_s - \frac{i \Delta r_s (\dot{\epsilon}_a - \dot{\epsilon}_s) \lambda_o}{4\pi \sqrt{\dot{\epsilon}_a} \cos(\theta)}$$

Approximation:
$$\langle \epsilon \rangle = \frac{\epsilon_s}{\epsilon_a} + d \frac{4\pi i \sqrt{\epsilon_a}}{\lambda} \frac{\epsilon_s (\epsilon_s - \epsilon_o) (\epsilon_o - \epsilon_a)}{\epsilon_o (\epsilon_s - \epsilon_a)} \sqrt{\frac{\epsilon_s}{\epsilon_a} - \sin^2 \phi_a}$$

$$\epsilon_o = \xi \pm (\xi^2 - \langle \epsilon \rangle \epsilon_a)^{1/2},$$

$$\xi = \frac{1}{2} (\langle \epsilon \rangle + \epsilon_a) + \frac{i\lambda (\langle \epsilon \rangle - \epsilon_a) \langle \epsilon \rangle'}{8\pi \langle n_z \rangle \langle \epsilon \rangle}.$$

The Role of Surfaces and Interfaces



“God made solids, but surfaces were the work of the Devil.” Wolfgang Pauli

- Material Science: Function strongly dependent on the surface/ interface properties: for sensors, semiconductor devices, etc.
- Broken symmetry -> dimers, image charges, modification of bulk properties, surface stress and surface strain!
- Intrinsic effects: band bending, surface phonons, surface plasmons,
- “Dirt effects:” roughness, diffusion, steps, Quantum dots
- When you can´t avoid it; HOW can you control/monitor it?