COHERENCE AND INTERFERENCE

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Interference is a topic in optics which undergraduate physics, optics and EE students already learn in their second year. Assuming that the electric field behaves as a plane wave, one can explain well the colors of thin coatings, diffraction effects of the type introduced by Huygens, the spectral response of Bragg gratings, and many more optical phenomena. We all learn (and teach) in undergraduate courses that one has to sum the electric field vectors. However, writing down a plane wave and using the mathematical techniques does usually not give the correct explanations for some experiments. Plane waves, behaving as $\overrightarrow{E}(\vec{r},t) = \vec{E}_0 e^{i\vec{k}.\vec{r}-i\omega t-\varphi}$, are solutions of Maxwell equations in any homogeneous region, but they do not grasp the physics of real wave trains or photons in two respects. Using them one assumes:

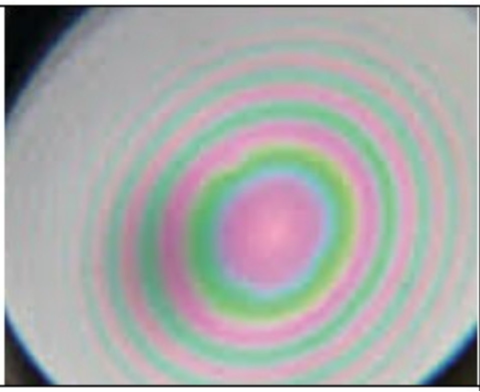
1) Along the propagation direction the field (photon) is infinitely long;

2) and perpendicular to the propagation direction the field (photon) is infinitely wide;

Despite for understanding some physical effects plane waves are helpful, there are two shortcomings inherently connected with them

1) There is no photon (or wave train) which has in infinitely long coherence length, but the expression above gives the impression that the phase is well defined for any instant and any point in space along the propagation direction. We all know from experience that each wave (photon) has a finite coherence length. The colorful pattern of a thin oil film on water or from a thickness variation of an oxide layer on silicon (see Fig. 1) arises from this effect; however, when asking students why a thicker oil film (> ~1 µm) does not show such interference colors anymore, many explanations are proposed, but seldom the correct one that the coherence length of light is already too small to yield interference (with our eyes as broadband detector and the photons from the sun as broadband source!).

2) Furthermore, there is also no wave field which has normal to the propagation direction a definite and well defined phase. Experimentally it turns out that the correlation of the fields vanishes also perpendicular to the propagation direction , which is shown with Young's double slit experiment if the two slits are far apart. This has been used by two astronomers Hanbury-Brown and Twiss to determine the size of stars in other galaxies.[1,2] In other words: even if strictly monochromatic light is used, each photon has also a finite coherence area $\Delta A = Q^2 = R^2 \lambda^2/\Delta S$ with the areal size of the light source ΔS, the distance R and the quasimonochromatic wavelength λ. The quantity $\sqrt{\Delta A}$ is also sometimes called coherence distance.



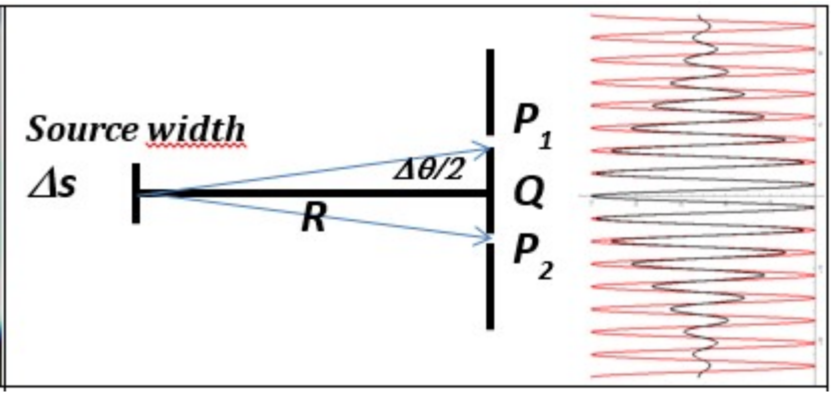


Fig.1: Newton's rings, produced through slight and continuous thickness variations of an oxide film on silicon. One can see the vanishing colour of the rings (i.e. temporal decoherence), which indicates that the coherence length of light is approximately of the same order than the oxide thickness.

Fig.2: Schematic sketch of Young's interference fringes. The red line shows the intensity variations (on a screen in the far field) calculated with assumed infinite coherence, the black line the true experimental result. The finite coherence area, yielding to a damping of interference fringes(i.e. spatial decoherence), is given by $\Delta A = Q^2 = R^2\lambda^2/\Delta S$, with ΔS as (vertical) extension of the light source, R, λ for the distance and monochromatic wavelength, yielding a finite coherence area ΔA.

ONLY INTENSITIES ARE MEASUREABLE QUANTITIES

Any optical detector for optical radiation never measures the field, all measure **intensities**! And because each intensity measurement takes long against the oscillation period of light $(\sim 10^{-15}s)$ always intensities are measured. (Perhaps fast sampling oscilloscopes work up to 10 GHz, but this is still far away from optical frequencies.) So, in the case of two interfering fields, e.g. from a Michelson interferometer, or from two different slits in Young's experiment, (respectively Huygen's principle), the measured intensity is always $I = \varepsilon_0\, n\, c \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \left(\vec{E}_1(t) + \vec{E}_2(t)\right)^2 dt$.

Provided $\vec{E}_1(t), \vec{E}_2(t)$ are deterministic and well defined quantities over time and space, the square of the sum can be formed straightforward yielding interference terms; however, if $\vec{E}_1(t), \vec{E}_2(t)$ are not well correlated any more due to a finite coherence length / distance, and their **mutual correlation** vanishes, then instead of coherence terms yielding a cosinusoidal intensity variation, the sum of intensities is measured. All the arguments above are only classical and I just note in passing that quantum optics yields $\Delta N \Delta\phi \geq 1$, with ΔN as standard deviation for the photon number, (which implies that beams with a definite photon number have a totally undefined phase).

EFFECT ON POLARIZATION

Until now we have just argued with scalar electrical fields and discussed for intensity measurements autocorrelation functions $< E_x(\vec{r}, t+\tau) E_x^*(\vec{r}\,', t) >)$ which manifest their correlation by the sharpness (visibility) of fringes in Michelson's (temporal) or Young's (spatial) interference experiments. Polarization measurements are usually done by measuring the (time averaged) correlations **of different components of the electric field at a single point in time and space** (loosely written as $< E_i(\vec{r}, t) E_j^*(\vec{r}, t) >$).. In optics, we do not measure the electric fields, since the available detectors are much too slow, but their statistical second moments (see the correlation function 2 lines above). Based on a proposal by J. Humlicek in chapter 1.4 of ref. 3, R. Ossikovski and K. Hingerl recently published[4,5] two papers how the effect of decoherence can be formulated analytically to describe the effect of decoherence on depolarization. For both cases, it turns out that the intensity measurement, or better the four intensity measurements to determine the Stokes parameters, can be predicted by 16 measurements (Müller Matrix elements) for all possible input and output polarization vectors.

References

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